BISIMULATION AND SIMULATION: ALGORITHMS AND APPLICATIONS PH.D. COURSE 2017/18

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Abstract. This is a preliminary program for a Ph.D. course whose aim is introducing, starting from very basic notions, the problem of computing bisimulation and simulation relations on finite structures (i.e. graphs).

Class 1: (A. Policriti — March 12th)

- Basic definitions: DFA (NFA) and Myhill-Nerode theorem.
- Hopcroft algorithm for minimization of the number of states of a DFA.
- Paige-Tarjan-Bonic algorithm for the single function coarsest partition problem.
- Hopcroft-Karp algorithm.

Basically we will deal with the deterministic case only. The uniqueness of the minimum automaton is proved from Myhill-Nerode. Following we have the introduction of top-down and bottom-up strategies, with related challenges and problems. Final considerations on Hopcroft-Karp algorithm and the language equivalence problem.

Class 2: (A. Policriti — March 15th)

- Minimization on a non-deterministic context. Minimizing with respect to (binary) *relations*.
- Automata and non well-founded sets: bisimulation in a set-theoretic context.
- Sets/Hypersets: $\mathsf{HF}^0, \mathsf{HF}^{1/2}$, and HF^1.
- Ackermann encoding (and variants).

We move to the non-deterministic case rewriting the bisimulation problem in relational terms and clarifying where and why previously presented techniques would not work. We then introduce the set-theoretic point of view starting from well-founded hereditarily finite sets (presented as acyclic graphs) and moving to the non well-founded case (that is, cyclic graphs). Equality and bisimulation are proved to capture the same relation. We conclude providing a further approach to set-equality computation, based on numerical encoding of sets (graphs).

Class 3: (A. Policriti — March 19th)

- Dovier-Piazza-Policriti algorithm: mixing the previously presented views on bisimulation computation.
- Labels and their elimination.
- ... [new results ?] ...

We consider a mixed bottom-up/top-down strategy resulting in an algorithm combining the coarsest relation computation with the set-theoretic view on bisimulation. We then look at labels on graphs and at the (many) way to get read of them.

Class 4: (R. Gentilini — April 4th)

- Simulation vs Bisimulation
- Simulation as a game (O(mn) algorithm by Rizzi et al.)
- Simulation as coarsest partition problem (Generalized coarsest partition problem).

We move from the bisimulation to the simulation problem. We present the notion of simulation and its applications for comparing and reducing structures. We presents two characterizations of simulation (in terms of games and as a generalized coarsest partition problem) leading to two algorithmic solutions for the corresponding problem.

Class 5: (R. Gentilini — April 6th)

- OBDDs and Symbolic Algorithms
- A symbolic algorithm for strongly connected components

We introduce symbolic algorithms, based on OBDD data structures, and their role to deal with large graphs. We illustrate a symbolic algorithm for computing strongly connected components (Gentilini-Piazza-Policriti SCC algorithm + analysis Chattarjee in his recent SODA paper)

Class 6: (R. Gentilini — April 9th)

- weighted automata (WA) and transducers
- WA and determinization
- WA and minimization

We move from classic to weighted automata. Unlike the unweighted case, some WA do not admit equivalent deterministic machines. We present the twin property (a sufficient condition for determinization) and Mohri algorithm to determinize WA enjoying the twin property. We present the recent results by Kupferman et. al. on rigorous approximated determinization of weighted automata. We present a generalization of Hopcroft algorithm to minimize deterministic WA.

Class 1: (C. Piazza — April 11th)

Hybrid automata and minimization (bisimulation and simulation): the infinite.

- Hybrid Automata: syntax
- Hybrid Automata: semantics
- Bisimulation and simulation on Hybrid Automata

We introduce the basic definitions of Hybrid Automata. Hybrid automata are a finite representation for infinite labeled transition systems. The high expressive of hybrid automata immediately leads to undecidability results. However, when bisimulation/simulation quotients reduce the infinite labeled transition system induced by a hybrid automaton to a finite graph, significant properties become decidable.

Class 2: (C. Piazza — April 13th)

Decidability/undecidability and computational complexity on HA.

- Classes of Hybrid Automata having finite bisimulation quotient
- Classes of Undecidable Hybrid Automata
- Tarsky's theorem for deciding Hybrid Automata
- Approximation algorithms/semantics

We describe in more details some classes of hybrid automata for which the bisimulation/simulation quotient can be proved to be finite. On the other hand, for other classes of hybrid automata undecidability results witness the infinite dimension of the quotients. We describe cases in which despite the simulation quotient is infinite other quotienting techniques derived from Tarsky's decidability results over semi-algebraic sets can be exploited to get decidability.

Class 3: (C. Piazza — April 16th)

Alternative view: Markov Chains and lumping (bisimulation and multi-sets).

- Continuous and Discrete Markov Chains
- The notion of lumpability
- Valmari-Franceschinis algorithm and generalizations

We introduce Continuous and Discrete Markov Chains together with some standard properties and examples of application. The notion of lumpability, in all its variants, is a generalization of the notion of bisimulation on Markov Chains. We give some intuitions on Valmari-Franceschinis algorithm which generalizes Paige-Tarjan one to lumpability case.

Invited lectures

2/3 invited lectures (one by Miculan)

Readings on selected topics

Presentations by attendees on selected topics.